

## THE CARPET RIDE PROBLEM

You are a young traveler, leaving home for the first time. Your parents want to help you on your journey, so just before your departure, they give you two gifts. Specifically, they give you two forms of transportation: a hover board and a magic carpet. Your parents inform you that both the hover board and the magic carpet have restrictions in how they operate:



We denote the restriction on the *hover board's* movement by the vector  $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ .

By this we mean that if the hover board traveled “forward” for one hour, it would move along a “diagonal” path that would result in a displacement of 3 miles East and 1 mile North of its starting location.



We denote the restriction on the *magic carpet's* movement by the vector  $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ .

By this we mean that if the magic carpet traveled “forward” for one hour, it would move along a “diagonal” path that would result in a displacement of 1 mile East and 2 miles North of its starting location.

### SCENARIO ONE: THE MAIDEN VOYAGE

Your Uncle Cramer suggests that your first adventure should be to go visit the wise man, Old Man Gauss. Uncle Cramer tells you that Old Man Gauss lives in a cabin that is 107 miles East and 64 miles North of your home.

#### TASK:

Investigate whether or not you can use the hover board and the magic carpet to get to Gauss’s cabin. If so, how? If it is not possible to get to the cabin with these modes of transportation, why is that the case? Justify your answer algebraically and/or geometrically.

### SCENARIO TWO: HIDE-AND-SEEK

Old Man Gauss wants to move to a cabin in a different location. You are not sure whether Gauss is just trying to test your wits at finding him or if he actually wants to hide somewhere that you can't visit him.

#### Are there some locations that he can hide and you cannot reach him with these two modes of transportation?

Describe the places that you can reach using a combination of the hover board and the magic carpet and those you cannot. Specify these geometrically and algebraically.

## THE CARPET RIDE PROBLEM: GETTING BACK HOME

Suppose you are now in a three-dimensional world for the carpet ride problem, and you have three modes of

transportation:  $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ ,  $\mathbf{v}_2 = \begin{bmatrix} 6 \\ 3 \\ 8 \end{bmatrix}$ ,  $\mathbf{v}_3 = \begin{bmatrix} 4 \\ 1 \\ 6 \end{bmatrix}$ .

You are only allowed to use each mode of transportation **once** (in the forward or backward direction) for a fixed amount of time ( $c_1$  on  $\mathbf{v}_1$ ,  $c_2$  on  $\mathbf{v}_2$ ,  $c_3$  on  $\mathbf{v}_3$ ). Find the amounts of time on each mode of transportation ( $c_1$ ,  $c_2$ , and  $c_3$ , respectively) needed to go on a journey that starts and ends at home OR explain why it is not possible to do so.